YALLA

Yet Another Logic Language for Argumentation

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INRA & IRIT, France

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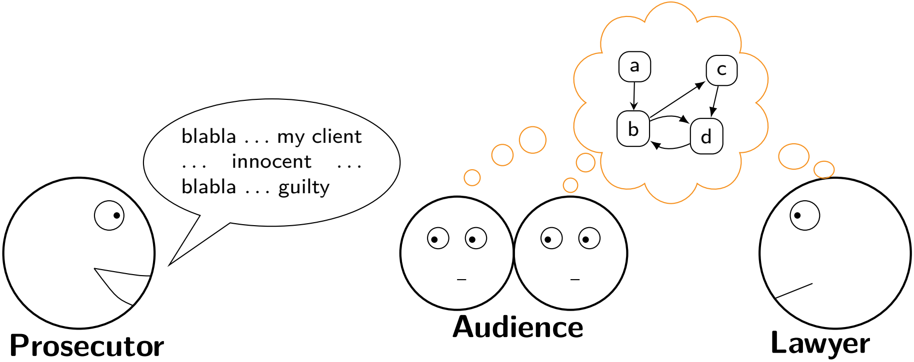
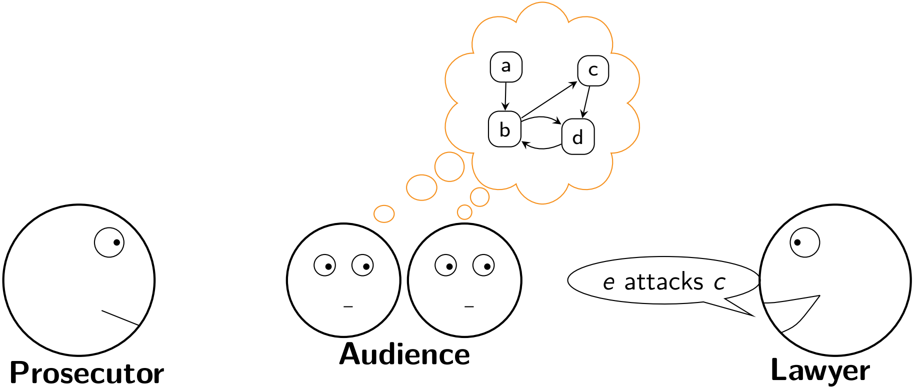
A Lawyer During a Trial



- A lawyer (the agent) is going to make her final address to an audience (the target).
- She knows (approximatively) the **argumentation system** (AS) of the target.

A lawyer during a trial



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- She has to make a change to the target AS:

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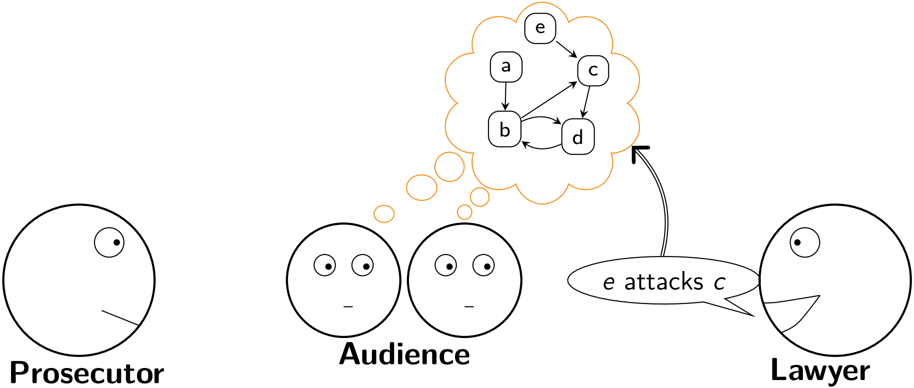

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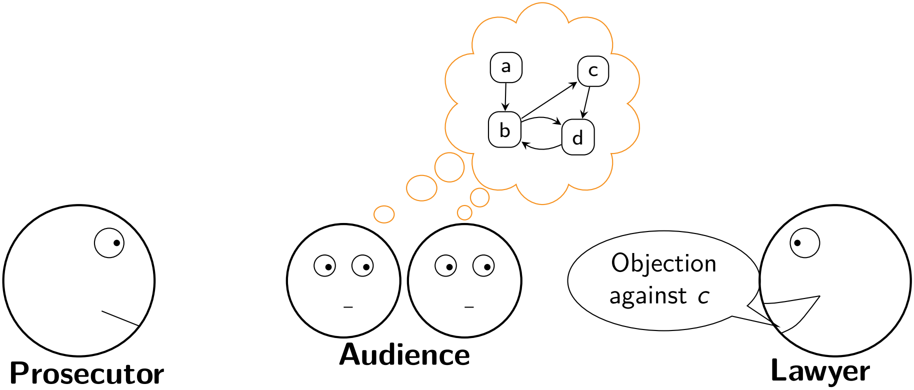

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  - or by doing an objection about an argument (to remove it).

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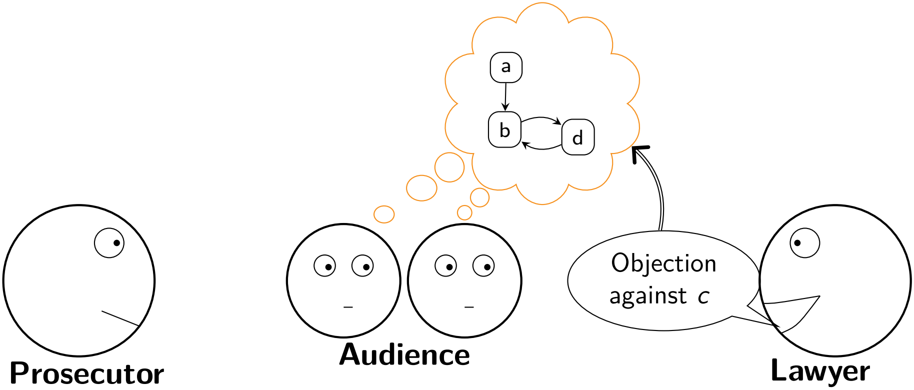

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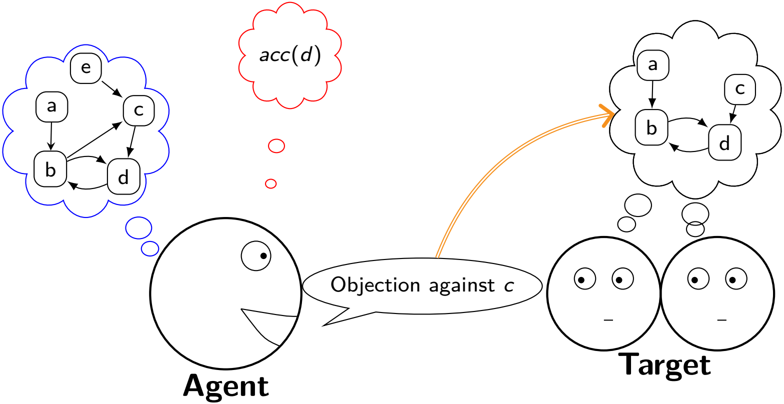


- Agent:
  - has a private argumentation system (her knowledge)
  - has a goal w.r.t. the target
  - should respect some constraints

    ⇒ notion of **executable operation**

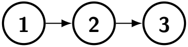
Outline

1. YALLA and Abstract Argumentation
   - Dung Framework
   - Semantics
2. YALLA and Argumentation Dynamics
3. Concluding Remarks

Dung framework

- According to Dung, an **abstract argumentation system** is a pair $(\mathcal{A}, \mathcal{R})$, where :
  - $\mathcal{A}$ is a finite nonempty set of *arguments* and
  - $\mathcal{R}$ is a binary relation on $\mathcal{A}$, called *attack relation*

- This system can be represented by a graph denoted $\mathcal{G}$



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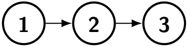

- YALLA: a **term** is a **set of arguments**
  - $singl(\{1\}) \wedge singl(\{2\}) \wedge singl(\{3\}) \wedge (\{1\} \rhd \{2\}) \wedge (\{2\} \rhd \{3\})$

Universe

A **universe** $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ = all arguments and their interactions.

0. Mr. X is not guilty of the murder of Mrs. X

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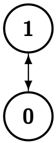

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2. Mr. X's business associate has sworn that he met him at the time of the murder.

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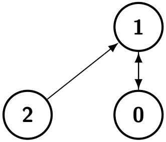

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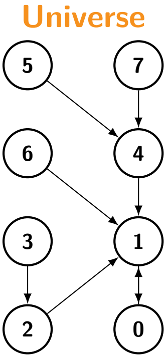
0. Mr. X is not guilty of the murder of Mrs. X
1. Mr. X is guilty of the murder of Mrs. X
2. Mr. X's business associate has sworn that he met him at the time of the murder.
3. Mr. X associate's testimony is suspicious due to their close working business relationship
4. Mr. X loves his wife. A man who loves his wife cannot be her killer.
5. Mr. X has a reputation for being promiscuous.
6. Mr. X had no interest to kill Mrs. X, since he was not the beneficiary of her life insurance
7. Mr. X is known to be venal and his "love" for a very rich woman could be only lure of profit.



Universe

A **universe** $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ = all arguments and their interactions.

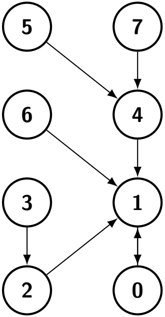
Definition (Argumentation graph)

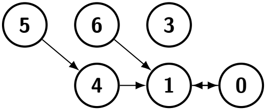
*An **argumentation graph** $\mathcal{G}$ is a pair $(\mathcal{A}, \mathcal{R})$*

- $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ *arguments (finite)*
- $\mathcal{R} \subseteq \mathcal{R}_{\mathbf{U}} \cap (\mathcal{A} \times \mathcal{A})$

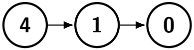
Γ = *all argumentation graphs w.r.t. the universe.*

Universe





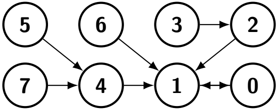
$\mathcal{G}_1$



$\mathcal{G}_2$

Argumentation Graph Example

- Agent L knows some of the arguments of the universe ($\mathcal{G}_L \subseteq \Gamma$):



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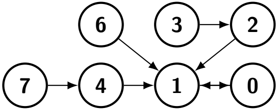

$\mathcal{G}_L$

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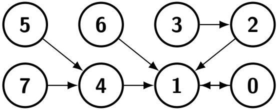

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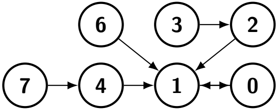
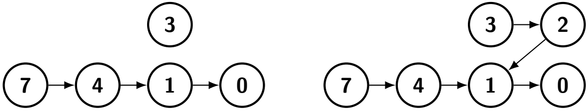


$\mathcal{G}_L$

$$
\begin{array}{ccccccc}
 & on(\{0,1,2,3,4,6,7\}) & & \wedge & \neg(on(\{5\})) & \wedge \\
(\{1\} \rhd \{0\}) & \wedge & (\{4\} \rhd \{1\}) & \wedge & \ldots & \wedge \\
\neg(\{5\} \rhd \{4\}) & & & & &
\end{array}
$$

Argumentation Graph Example

- But L *is not sure* about the content of the jury's system. She hesitates between two graphs:



$\mathcal{G}_{J1}$ $\mathcal{G}_{J2}$

$$
\begin{array}{ccccc}
 & on(\{0,1,3,4,7\}) & & \wedge & \\
\neg(on(\{5\})) & \wedge & \neg(on(\{6\})) & \wedge & \ldots \quad \wedge \\
\left(\begin{array}{c} (\neg(on(\{2\})) \;\wedge\; \neg(\{2\} \rhd \{1\}) \;\wedge\; \neg(\{3\} \rhd \{2\})) \\ \vee \\ (on(\{2\}) \;\wedge\; (\{2\} \rhd \{1\}) \;\wedge\; (\{3\} \rhd \{2\})) \end{array} \right) & & & &
\end{array}
$$

Outline

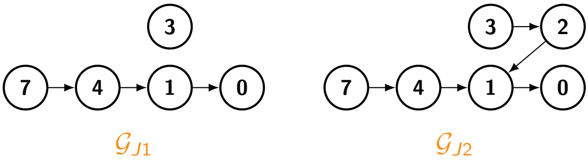
Semantics Criteria

- A set $\mathcal{S}$ is **conflict-free** iff there do not exist $a, b \in \mathcal{S}$ such that a attacks b
  - $F(t) \iff on(t) \wedge (\neg(t \rhd t))$
- $\mathcal{S}_1$ **defends** each argument of $\mathcal{S}_2$ iff each attacker of an argument of $\mathcal{S}_2$ is attacked by an argument of $\mathcal{S}$
  - $t_1 \ \ggg t_2 \iff (\forall t_3\ ((singl(t_3) \wedge (t_3 \rhd t_2)) \implies (t_1 \rhd t_3)))$
- $\mathcal{S}$ is an **admissible** set iff it is conflict-free and it defends all its elements
  - $A(t) \iff (F(t) \wedge (t \ \ggg t))$

Acceptability Semantics

- $\mathcal{E}$ is a **complete extension** iff $\mathcal{E}$ is an admissible set and every acceptable argument wrt $\mathcal{E}$ belongs to $\mathcal{E}$
  - $C(t) \iff (A(t) \land \forall t_2 \, ((singl(t2) \land (t \;\, \rhd\!\!\!\rhd \, t_2)) \implies (t_2 \subseteq t)))$
- $\mathcal{E}$ is the **only grounded extension** iff $\mathcal{E}$ is the smallest complete extension
  - $G(t) \iff (C(t) \land \forall t_2 \, (C(t_2) \implies (t \subseteq t_2)))$

Argumentation Graph Example



$$
\begin{array}{ccccccc}
 & & on(\{0,1,3,4,7\}) & & \wedge & & \\
\neg(on(\{5\})) & \wedge & \neg(on(\{6\})) & & \wedge & \dots & \wedge \\
\{7\} \gg \{1\} & \wedge & G(\{1,3,7\}) & & & &
\end{array}
$$

Outline

Change in Argumentation

- ([Cayrol et al., 2010]): four **elementary** change operations.
  - adding/removing an argument with related attacks,
  - adding/removing an attack.
- Modification to handle multi-agents scenario

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  - adding/removing an argument with related attacks,
  - ~~adding/removing an attack.~~

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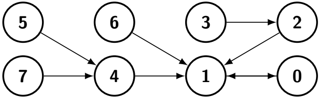
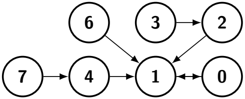
Executable Operations: Example

- Given the universe:



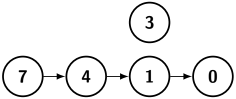
$(\ominus, 2, \varnothing)$, $(\ominus, 4, \varnothing)$, $(\oplus, 5, \{(5,4)\})$ and $(\oplus, 6, \{(6,1)\})$ are **elementary operations**

- With $\mathcal{G}_L$:



$(\ominus, 2, \varnothing)$, $(\ominus, 4, \varnothing)$ and $(\oplus, 6, \{(6,1)\})$ are **allowed** for Agent L (arguments she knows).

- On the target $\mathcal{G}_{J1}$:



$(\ominus, 4, \varnothing)$ and $(\oplus, 5, \{(5,4)\})$ are **executable** by L on G_{J1}.

Parallel

- An agent may **act on** a target argumentation system
- An agent has a **goal**
- An agent has access to some **transitions**

  $\Rightarrow$ Close to **belief update**

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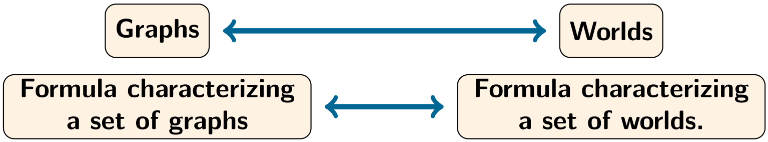


	Arg. Change	Update
Initial knowledge:	Set of AS	Set of worlds
Input:	Goal	New info
Constraints:	Set of transitions (executable operations)	None (every update is achievable)

Outline

Update Operator Related to Authorized Transitions

- **Change** in argumentation **is close to** **authorized transitions** in belief change

$\Rightarrow$ Belief update with **authorized transitions**

- **Modification** of the belief update **postulates** to account for transition constraints
- Introduction of a **new representation theorem** linking these postulates to a preorder on graphs

Preferences of the Lawyer

- The lawyer may have preferences over the transitions
  - if $(\mathcal{G}, \mathcal{G}_i) \in \mathcal{T}_+$ and $(\mathcal{G}, \mathcal{G}_j) \in \mathcal{T}_-$, then $\mathcal{G}_i \prec_{\mathcal{G}} \mathcal{G}_j$
  - else, if $(\mathcal{G}, \mathcal{G}_i) \in \mathcal{T}_e$ and $(\mathcal{G}, \mathcal{G}_j) \notin \mathcal{T}_e$, then $\mathcal{G}_i \prec_{\mathcal{G}} \mathcal{G}_j$

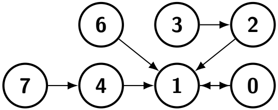
Where

$$\mathcal{T}_+ = \left\{ (\mathcal{G}_1, \mathcal{G}_2) \;\middle|\; \begin{array}{l} \exists o \text{ such that } o \text{ is an } \textbf{addition} \text{ executable on } \mathcal{G}_1 \text{ and} \\ \mathcal{G}_2 = o(\mathcal{G}_1) \end{array} \right\}$$

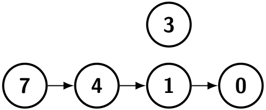
$$\mathcal{T}_- = \left\{ (\mathcal{G}_1, \mathcal{G}_2) \;\middle|\; \begin{array}{l} \exists o \text{ such that } o \text{ is a } \textbf{removal} \text{ executable on } \mathcal{G}_1 \text{ and} \\ \mathcal{G}_2 = o(\mathcal{G}_1) \end{array} \right\}$$

$$\mathcal{T}_e = \mathcal{T}_+ \cup \mathcal{T}_-$$

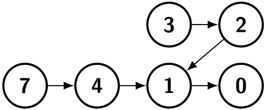
Preferences of the Lawyer



$\mathcal{G}_L$



$\mathcal{G}_{J1}$



$\mathcal{G}_{J2}$

$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{J1})$	$(\oplus, a_2, \{(a_2, a_1), (a_3, a_2)\})(\mathcal{G}_{J1})$
$(\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{J1})$	$(\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{J2})$
$(\ominus, a_0, \varnothing)(\mathcal{G}_J)$	$(\ominus, a_1, \varnothing)(\mathcal{G}_J)$
$(\ominus, a_3, \varnothing)(\mathcal{G}_J)$	$(\ominus, a_4, \varnothing)(\mathcal{G}_J)$
$(\ominus, a_7, \varnothing)(\mathcal{G}_J)$	
$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{J2})$	$(\oplus, a_2, \{(a_2, a_1), (a_3, a_2)\})(\mathcal{G}_{J2})$
$(\ominus, a_2, \varnothing)(\mathcal{G}_J)$	$(\ominus, a_5, \varnothing)(\mathcal{G}_J)$
...	

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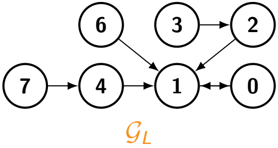
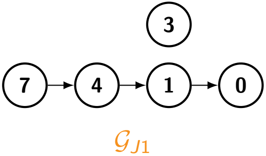
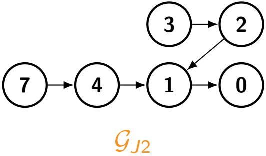


Goal: $\exists p, G(p) \wedge (\{0\} \subseteq p)$





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$(\ominus, a_3, \varnothing)(\mathcal{G}_J)$	$(\ominus, a_4, \varnothing)(\mathcal{G}_J)$
$(\ominus, a_7, \varnothing)(\mathcal{G}_J)$	
$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{J2})$	$(\oplus, a_2, \{(a_2, a_1), (a_3, a_2)\})(\mathcal{G}_{J2})$
$(\ominus, a_2, \varnothing)(\mathcal{G}_J)$	$(\ominus, a_5, \varnothing)(\mathcal{G}_J)$
...	

Outline

Change Characterizations

- A characterization gives **necessary and/or sufficient conditions** to obtain a particular goal, given
  - an operation type,
  - a semantics.

When adding an argument z under the grounded semantics, if z is not attacked by $\mathcal{G}$, z indirectly defends x and $x \notin \mathcal{E}$, then $x \in \mathcal{E}'$.

- If the conditions are met, then the conclusion is true on the system after the change.

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$\Rightarrow$ Used to find a way to achieve the goal.

Specific Update Postulates

- **Characterizations $\rightarrow$ update postulates**
- "When adding an argument z under the grounded semantics, if z is not attacked by $\mathcal{G}$, z indirectly defends x and $x \notin \mathcal{E}$, then $x \in \mathcal{E}'$."

$\Rightarrow$ Corresponding postulate:

let $\quad \mathcal{G} = (\mathcal{A}, \mathcal{R}_\mathcal{A})$ and $o = \langle \oplus, z, \mathcal{R}_z \rangle$,

if $\quad \mathcal{G} \models G(p) \wedge singl(x) \wedge \neg(x \subseteq p)$ and $(\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e$ and
$o(\mathcal{G}) \models (singl(z) \wedge \neg(\exists t\ on(t) \wedge (t \rhd z)) \wedge (z \rhd\!\!\longrightarrow\!\!\rhd x))$

then $\quad \mathcal{G} \diamondsuit_\mathcal{T} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models G(p') \wedge (x \subseteq p')$.

Outline

Conclusion

- A unified view of dynamics in argumentation with
  - a language (YALLA) that captures all the notions of abstract argumentation domain
  - a set of postulates specific for argumentation dynamics
- Possibility to represent
  - the knowledge of an agent as an argumentation system and her goals
  - how she can interact with a target argumentation system (stage of a debate).

Future Works

- Relax the executability constraint
  - Adding an argument that is already present
  - Removing an argument that is not there
- Analyse the link between non elementary operations (simultaneously addition/removal of arguments) and a sequence of elementary operations
- Study the evolution of a private argumentation system with belief revision
  - An agent thinks that x is rejected and someone informs her that it is not the case

  $\rightarrow$ is there an argument that defends x?

  $\rightarrow$ is the attacker of x valid?

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  - An agent thinks that x is rejected and someone informs her that it is not the case
  - $\rightarrow$ is there an argument that defends x?
  - $\rightarrow$ is the attacker of x valid?

- Thank you!

References I

Cayrol, C., Dupin de Saint Cyr, F., and Lagasquie-Schiex, M.-C. (2010).
Change in abstract argumentation frameworks: Adding an argument.
Journal of Artificial Intelligence Research, 38:49–84.

Executable Operation I

$(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ = universe, k = agent, $\mathcal{G}_k = (\mathcal{A}_k, \mathcal{R}_k)$ her AS,
$\mathcal{G} = (\mathcal{A}, \mathcal{R})$ any AS.

- **elementary operation** $o = (op, x, att)$
  $op \in \{\oplus, \ominus\}, \quad x \in \mathcal{A}_{\mathbf{U}}, \quad att \subseteq \mathcal{R}_{\mathbf{U}}$ and
  - $op = \oplus$: $\forall (u, v) \in att,\ (u \neq v)$ and $(u = x \textit{ or } v = x)$
  - $op = \ominus$: $att = \varnothing$

- (op, x, att) **allowed for** k iff $x \in \mathcal{A}_k$ and $att \subseteq \mathcal{R}_k$

Executable Operation II

$(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ = universe, k = agent, $\mathcal{G}_k = (\mathcal{A}_k, \mathcal{R}_k)$ her AS,
$\mathcal{G} = (\mathcal{A}, \mathcal{R})$ any AS.

- (op, x, att) **executable by k on $\mathcal{G}$** iff:
  - $op = \oplus$: $x \notin \mathcal{A}$ and $\forall (u, v) \in att,\ (u \in \mathcal{A}\ or\ v \in \mathcal{A})$
  - $op = \ominus$: $x \in \mathcal{A}$.

- $o = (op, x, att)$ executable by k on $\mathcal{G}$ **provides** a new system $\mathcal{G}' = o(\mathcal{G}) = (\mathcal{A}', \mathcal{R}')$:
  - $op = \oplus$: $\mathcal{G}' = (\mathcal{A} \cup \{x\}, \mathcal{R} \cup \{att\})$
  - $op = \ominus$: $\mathcal{G}' = (\mathcal{A} \setminus \{x\}, \mathcal{R} \setminus \{(u, v) \in \mathcal{R} | u = x\ or\ v = x\})$

Signature and Structure

- **Signature** $\Sigma_{\mathbf{U}} = (V_{const}, V_f, V_P)$ where:
  - $V_{const} = \{c_\perp, c_1, \ldots, c_p\}$ $(p = 2^k - 1)$,
  - $V_f = \{union^2\}$,
  - $V_P = \{on^1, \rhd^2, \subseteq^2\}$.
- **Structure** $\mathcal{M} = (\mathcal{D}, \mathcal{I})$ of $\Sigma_{\mathbf{U}}$, associated with $(\mathcal{A}, \mathcal{R})$, where $\mathcal{D} = 2^{\mathcal{A}_{\mathbf{U}}}$ and $\mathcal{I}$ associates:
  - a unique element of $\mathcal{D}$ to each c_i
  - the union operator $(\mathcal{D}^2 \mapsto \mathcal{D})$ to *union*
  - the characterization of the subsets of $\mathcal{A}$ to *on* $(on(\mathcal{S})$ iff $\mathcal{S} \subseteq \mathcal{A})$
  - the inclusion relation to $\subseteq$
  - the attack between sets of arguments $(\mathcal{S}_1 \mathcal{R} \mathcal{S}_2$ iff $\mathcal{S}_1 \subseteq \mathcal{A}, \mathcal{S}_2 \subseteq \mathcal{A}$ and $\exists x_1 \in \mathcal{S}_1, x_2 \in \mathcal{S}_2, (x_1 \mathcal{R} x_2))$ to $\rhd$

Axioms I

- Axioms for set inclusion
  - $\forall x \ (c_\perp \subseteq x)$
  - $\forall x \ (x \subseteq x)$
  - $\forall x, y, z \ ((x \subseteq y \land y \subseteq z) \implies x \subseteq z)$.
- Axioms for set operator
  - $\forall x, y \ ((x \subseteq union(x, y))$
  - $\forall x, y \ ((y \subseteq union(x, y))$
  - $\forall x, y, z \ (((x \subseteq z) \land (y \subseteq z)) \implies (union(x, y) \subseteq z)))$

Axioms II

- Axioms combining set-inclusion and attack relation
  - $\forall x, y, z\ (((x \rhd y) \wedge (x \subseteq z)) \implies (z \rhd y))$
  - $\forall x, y, z\ (((x \rhd y) \wedge (y \subseteq z)) \implies (x \rhd z))$
  - $\forall x, y, z\ ((union(x, y) \rhd z) \implies ((x \rhd z) \vee (y \rhd z)))$
  - $\forall x, y, z\ ((x \rhd union(y, z)) \implies ((x \rhd y) \vee (x \rhd z)))$
- Axioms for the predicate *on*:
  - $on(c_\perp)$
  - $\forall x, y\ ((on(x) \wedge (y \subseteq x)) \implies on(y))$
  - $\forall x, y\ ((on(x) \wedge on(y)) \implies on(union(x, y))$
  - $\forall x, y\ ((x \rhd y) \implies (on(x) \wedge on(y)))$

Useful Notations

- Let t_1 and t_2 be terms of $\text{YALLA}_{\mathbf{U}}$. We define:

$$t_1 = t_2 \stackrel{\text{def}}{\equiv} (t_1 \subseteq t_2) \wedge (t_2 \subseteq t_1)$$

$$t_1 \neq t_2 \stackrel{\text{def}}{\equiv} \neg(t_1 = t_2)$$

$$\begin{aligned} singl(t_1) \stackrel{\text{def}}{\equiv} & (t_1 \neq c_\perp) \wedge \\ & \forall t_2\ (((t_2 \neq c_\perp) \wedge (t_2 \subseteq t_1)) \implies (t_1 \subseteq t_2)) \end{aligned}$$

Formulae Expressing Semantics Criteria

- Let $\mathcal{A}_{\mathbf{U}}$ be a set of arguments, and $(\mathcal{A}, \mathcal{R})$ an AS such that $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. Let t, t_1, t_2, t_3 be terms of YALLA$_{\mathbf{U}}$:
  - **Conflict-freeness**: t is conflict-free in $(\mathcal{A}, \mathcal{R})$ iff
    $(\mathcal{A}, \mathcal{R}) \models on(t) \wedge (\neg(t \rhd t))$
    $\Longrightarrow F(t)$.
  - **Defense**: t_1 defends each element of t_2 in $(\mathcal{A}, \mathcal{R})$ iff
    $(\mathcal{A}, \mathcal{R}) \models (\forall t_3 \; ((singl(t_3) \wedge (t_3 \rhd t_2)) \implies (t_1 \rhd t_3)))$
    $\Longrightarrow (\mathcal{A}, \mathcal{R}) \models t_1 \;\rhd\!\!\rhd\; t_2$
  - **Admissibility**: t is admissible in $(\mathcal{A}, \mathcal{R})$ iff
    $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge (t \;\rhd\!\!\rhd\; t))$
    $\Longrightarrow (\mathcal{A}, \mathcal{R}) \models A(t)$

Formulae Expressing Semantics

- **Complete**: t is a complete extension of $(\mathcal{A}, \mathcal{R})$ iff $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2 \; ((singl(t2) \wedge (t \;\; \rhd\!\!\rhd t_2)) \implies (t_2 \subseteq t)))$ $\implies (\mathcal{A}, \mathcal{R}) \models C(t)$.
- **Grounded**: t is the grounded extension of $(\mathcal{A}, \mathcal{R})$ iff $(\mathcal{A}, \mathcal{R}) \models (C(t) \wedge \forall t_2 \; (C(t_2) \implies (t \subseteq t_2)))$ $\implies (\mathcal{A}, \mathcal{R}) \models G(t)$.
- **Stable**: t is a stable extension of $(\mathcal{A}, \mathcal{R})$ iff $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge \forall t_2 \; ((singl(t_2) \wedge \neg(t_2 \subseteq t)) \implies (t \rhd t_2)))$ $\implies (\mathcal{A}, \mathcal{R}) \models S(t)$.
- **Preferred**: t is a preferred extension of $(\mathcal{A}, \mathcal{R})$ iff $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2 \; (((t_2 \neq t) \wedge (t \subseteq t_2)) \implies \neg A(t_2)))$ $\implies (\mathcal{A}, \mathcal{R}) \models P(t)$.

Postulates Respecting Transition Constraints

✓ **U1:** $\varphi \diamondsuit_{\mathcal{T}} \alpha \models \alpha$

∼ **U2:** $\varphi \models \alpha \Rightarrow [\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\varphi]$ (optional: inertia)

✗ **U3:** $[\varphi] \neq \varnothing$ and $[\alpha] \neq \varnothing \Rightarrow [\varphi \diamond \alpha] \neq \varnothing$ (transition constraints)

$\Rightarrow$ **E3:** $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing$ iff $(\varphi, \alpha) \models \mathcal{T}$

✓ **U4:** $[\varphi] = [\psi]$ and $[\alpha] = [\beta] \Rightarrow [\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\psi \diamondsuit_{\mathcal{T}} \beta]$

✗ **U5:** $(\varphi \diamond \alpha) \wedge \beta \models \varphi \diamond (\alpha \wedge \beta)$ (enforcement failure)

$\Rightarrow$ **E5:** if $card([\varphi]) = 1$ then $(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta \models \varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)$

✗ **U8:** $[(\varphi \vee \psi) \diamond \alpha] = [(\varphi \diamond \alpha) \vee (\psi \diamond \alpha)]$ (enforcement failure)

$\Rightarrow$ **E8** if $([\varphi] \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$ or $([\psi] \neq \varnothing$ and $[\psi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$
then $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$
else $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = [(\varphi \diamondsuit_{\mathcal{T}} \alpha) \vee (\psi \diamondsuit_{\mathcal{T}} \alpha)]$

✓ **U9:** if $card([\varphi]) = 1$ then
$[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta] \neq \varnothing \Rightarrow \varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta) \models (\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta$

Representation Theorem

- **Assignment respecting** $\mathcal{T}$: $\forall \mathcal{G}_1, \mathcal{G}_2 \in \Gamma$ if $(\mathcal{G}, \mathcal{G}_1) \in \mathcal{T}$ and $(\mathcal{G}, \mathcal{G}_2) \notin \mathcal{T}$ then $\mathcal{G}_1 \prec_{\mathcal{G}} \mathcal{G}_2$.

Theorem

$\exists$ an operator $\diamondsuit_{\mathcal{T}}$ satisfying (**U1**,) **E3**, **U4**, **E5**, **E8**, **U9** iff
$\exists$ an assignment respecting $\mathcal{T}$ s.t. $\forall \mathcal{G} \in \Gamma, \forall \varphi, \alpha \in \mathsf{YALLA}_{\mathbf{U}}$,

- $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \left\{ \begin{array}{l} \mathcal{G}_1 \in [\alpha] \text{ such that } (\mathcal{G}, \mathcal{G}_1) \in \mathcal{T} \text{ and} \\ (\forall \mathcal{G}_2 \in [\alpha] \text{ such that } (\mathcal{G}, \mathcal{G}_2) \in \mathcal{T}, \mathcal{G}_1 \preceq_{\mathcal{G}} \mathcal{G}_2) \end{array} \right\}$
- $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ if $\exists \mathcal{G} \in [\varphi]$ such that $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$
- $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \bigcup_{\mathcal{G} \in [\varphi]} [\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha]$ otherwise